

What the Heck is a Distribution? A Survey for the Non-Analyst

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1 Abstract

“The Dirac delta is not a function, it’s a distribution, but that’s beyond the scope of this course” is a phrase uttered by nearly every undergraduate ODE instructor in existence since the Laplace transform has been taught. Since distribution theory is not commonly studied by non-analysts, this response can be very convenient (and appropriate) for instructors who don’t really know the formal justification to answer questions such as “What is the Dirac delta, really?” Wouldn’t it be nice to not be full of it and, at the very least, actually know what that statement means if you ever have to teach such a class? That was rhetorical, and the answer is yes. In this talk, I will discuss the fundamental properties of Schwartz functions and their continuous dual, the (tempered) distributions. I will also introduce the Fourier transform on these spaces and emphasize their roles in linear PDE theory. Some special attention will be paid to the Dirac delta throughout.