LOCAL ENERGY DECAY FOR THE DAMPED WAVE EQUATION

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Abstract: Local energy decay (LED) is a powerful estimate in dispersive PDE theory which heuristically expresses that the energy of a wave must decay quickly enough within every compact spatial set to be integrable in time. While solutions to the wave equation on Minkowski space always enjoy this property, the presence of non-trivial background geometry yields complications. One of the primary obstructions to LED is a geometric feature called *geodesic trapping*, which occurs when there are null geodesics which cannot exit a compact set. Bouclet and Royer proved that damped waves, which possess an intrinsic friction mechanism, still achieve LED on asymptotically Euclidean manifolds under the assumption of *geometric control* to handle trapped trajectories. After reviewing the history of LED for waves and damped waves, we will discuss new work which establishes LED for damped waves on asymptotically flat space-times which satisfy geometric control. The non-product manifold structure generates problematic error terms in the high frequency modes, which is also where the trapping takes effect. In this regime, we recover loss-less estimates by combining the strategies of Metcalfe, Sterbenz, and Tataru (who worked on non-trapping space-times but allowed for full Lorentzian structure) and Bouclet and Royer.